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CONSTRUCTURE OF UNIMODULAR SEQUENCES FOR WAVEFORM-AGILE SENSING

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Final Report

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14. ABSTRACT

Construction of complex-valued unimodular stochastic sequences or waveforms whose autocorrelation can be made arbitrarily small outside the origin has been done. This has been accomplished both for discrete and continuous waveforms. Besides, multidimensional waveforms having such low autocorrelation properties has also been constructed. The unimodular low autocorrelation stochastic waveforms constructed have then been characterized in terms of mathematical objects called frames. The closeness of such stochastic frames to orthonormal bases has also been studied.

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Significant accomplishments:

1. The main goal of constructing *complex-valued unimodular stochastic* waveforms (both discrete and continuous) whose autocorrelation can be made arbitrarily small outside the origin has been completed.

    - In the discrete case, this is first done using Gaussian random variables and then generalized to other random variables. The random variables that can be used for the construction of such waveforms has been characterized.
    - For the continuous case, the construction of such waveforms has been done using Brownian motion.
    - The construction of multidimensional waveforms having low autocorrelation has also been done.

2. Frames are now a standard tool in signal processing due to their effectiveness in robust signal transmission and reconstruction. The unimodular stochastic waveforms with low autocorrelation as described in 1. above has been characterized in terms of frames. Further analysis on

how close such frames are to being tight, i.e., how closely they resemble orthonormal bases has also been done.

3. Welch bounds are a family of lower bounds on the maximal cross correlation between vectors or signals. These are heavily used in signal processing in analyzing the best of the worst case behavior when the number of signals accessing a channel is more than the dimension of the channel and the desired orthogonality among the vectors cannot be maintained. Under this project, this idea has been extended to obtain lower bounds on the maximal cross correlation among *subspaces* of a given Hilbert space. These are expected to be useful in light of the emerging area of fusion frames.

4. An explicit construction of a tight frame from a given frame that avoids inverting the frame operator has been done. The tight frame obtained spans the same subspace as the original frame. In addition, the tight frame vectors are linear combinations of the original frame vectors and therefore the signal coefficients do not have to recomputed. This is particularly useful in areas of medical imaging where samples can only be taken on a set that may not be a tight frame and reconstruction time and error are both very important.

Archival publications during reporting period:

- S. Datta, *Construction of zero autocorrelation stochastic waveforms*, Advances in Pure Mathematics, Vol. 2., No. 6, pp. 428 - 440, Nov.

2012

- E. Au-Yeung & S. Datta, *Tight frames in spiral sampling*, Proc. 10th International Conf. on Sampling Theory and Applications (SampTA), Bremen, Germany, July 2013.

- E. Au-Yeung & S. Datta, *Tight frames, partial isometries, & signal reconstruction*, submitted.

- S. Datta, *Welch Bounds for Cross Correlation of Subspaces*, submitted.

Changes in research objectives, if any: none

Change in AFOSR program manager, if any: none

Extensions granted or milestones slipped, if any: none